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ALL RELATIVISTIC TRAJECTORIES OF BODIES ARE GEODESIC – *ET CETERA*

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SUNTO. — Si dimostra in vari modi che in relatività generale le traiettorie dei corpi sono sempre geodetiche, e pertanto non generano onde gravitazionali. Si dimostra poi che le soluzioni delle equazioni di Einstein a carattere ondulatorio non hanno alcuna reale esistenza fisica. Si dimostra infine la totale inaffidabilità dei modelli inflazionari. In Appendice alcune osservazioni complementari.

I dati sperimentali e osservativi confermano le nostre deduzioni.

ABSTRACT. — We prove in various ways that in general relativity (GR) the trajectories of the bodies are always geodesic — and therefore no gravitational wave (GW) is emitted. Then, we prove that the solutions of the Einstein field equations with a wave character do not have any physical existence. We prove finally the full unreliableness of the inflationary models. In Appendix some complementary remarks.

The experimental and observational data corroborate our deductions.

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1. – In sects. **2**÷ **5** we give several demonstrations that all the relativistic trajectories of bodies are *geodesic*. This property represents an evident proof that the gravitational waves (GWs) do not have a physical existence. The solutions of Einstein field equations with a wave character are only formal undulations quite destitute of a physical reality; various demonstrations of this fact are exposed in sects. **6**÷ **9**. In sect. **10** a proof of the unreliableness of the inflationary models.

The experimental and observational data (sects. **11** and **12**) corroborate our conclusions. The widespread, but erroneous, belief in the real-

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ity of the GWs has its origin in a superficial, but false, analogy with the electromagnetic waves.

In *Appendix* some complementary remarks.

2. – According to a clever treatment of the EIH-method developed by Infeld [1], we write, first of all, the Einstein field equations employing tensor densities:

$$(1)\quad \mathcal{R}_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\mathcal{R} = -8\pi\mathcal{T}_{\mu\nu}\ ; \quad (\mu,\nu = 0,1,2,3)\ ; \quad (c = G = 1)\ ;$$

$$(1')\quad \mathcal{R}_{\mu\nu} =: \sqrt{-g}\, R_{\mu\nu} \quad ; \quad \mathcal{R} =: \sqrt{-g}\, R \quad ; \quad \mathcal{T}_{\mu\nu} =: \sqrt{-g}\, T_{\mu\nu} \quad .$$

If $\delta(\mathbf{x}-\xi) =: \delta(x^1-\xi^1)\delta(x^2-\xi^2)\delta(x^3-\xi^3)$ is the three-dimensional Dirac's generalized function, and ξ^1, ξ^2, ξ^3 are the space coordinates of a particle, we write for a "cloud" composed of s particles:

$$(2)\qquad \mathcal{T}^{\mu\nu} = \sum_{p=1}^{s} \overset{p}{\mathcal{T}}{}^{\mu\nu} \quad ,$$

$$(3)\qquad \overset{p}{\mathcal{T}}{}^{\mu\nu} = \overset{p}{m}(t)\, \frac{\mathrm{d}\,\xi^{p\ \mu}}{\mathrm{d}\tau}\, \frac{\mathrm{d}\,\xi^{p\ \nu}}{\mathrm{d}\tau}\, \overset{p}{\delta}(\mathbf{x}-\xi) \quad ;$$

now, if the world-lines of the particles never intersect, it is not difficult to verify that the differential equations of motion of the particles:

$$(4)\qquad \sum_{p=1}^{s} \overset{p}{\mathcal{T}}{}^{\mu\nu}{}_{;\nu} = 0 \quad , \quad (\mu = 0,1,2,3) \quad ,$$

where the semicolon denotes a covariant derivative, give the equations of *geodesic* lines. No limitation exists for the values of the kinematical elements (velocities, accelerations, time derivative of the accelerations, *etc.*). The transition from a discrete to a continuous "dust" implies simply the substitution:

$$(5)\qquad \mathcal{T}^{\mu\nu} \rightarrow \varrho(t,\mathbf{x})\, \frac{\mathrm{d}\xi^{\mu}(t,\mathbf{x})}{\mathrm{d}\tau}\, \frac{\mathrm{d}\xi^{\nu}(t,\mathbf{x})}{\mathrm{d}\tau}\, \sqrt{-g} \quad ,$$

where ϱ is the mass density. –

We see that the gravitational self-force theory, which is based on a conceptually wrong analogy with the electromagnetic self-force of a charge, does not make any sense.

A last remark: in the *linearized* version of GR, the "dust" particles describe *straight* lines in a Minkowskian spacetime.

3. – Let us consider a system of non-interacting bodies moving in a Minkowskian spacetime. If $q^\mu(\tau)$, $\mu = 0, 1, 2, 3)$, are the translational coordinates of one of them as functions of proper time τ, we have that

$$\mathcal{L}_{(0)} := \eta_{\mu\nu} \frac{\mathrm{d}q^\mu}{\mathrm{d}\tau} \frac{\mathrm{d}q^\nu}{\mathrm{d}\tau} = c^2 \tag{6}$$

is a first integral of Lagrange equations

$$\frac{\partial \mathcal{L}_{(0)}}{\partial q^\mu} - \frac{\partial}{\partial \tau} \frac{\partial L_{(0)}}{\partial \left(\mathrm{d}q^\mu/\mathrm{d}\tau\right)} = 0 \quad , \tag{7}$$

from which:

$$\frac{\mathrm{d}^2 q^\mu}{\mathrm{d}\tau^2} = 0 \quad , \tag{8}$$

i.e. a rectilinear and uniform motion.

Quite analogously, if we consider a system of bodies interacting *only* gravitationally and moving in the Riemann-Einstein manifold created by them, we have that

$$\mathcal{L} := g_{\mu\nu}\left[q(\tau)\right] \frac{\mathrm{d}q^\mu}{\mathrm{d}\tau} \frac{\mathrm{d}q^\nu}{\mathrm{d}\tau} = c^2 \tag{9}$$

is a first integral of Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial q^\mu} - \frac{\partial}{\partial \tau} \frac{\partial L}{\partial \left(\mathrm{d}q^\mu/\mathrm{d}\tau\right)} = 0 \quad , \tag{10}$$

which coincides with the *geodesic* equations

$$\frac{\mathrm{d}^2 q^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu{}_{\varrho\sigma} \frac{\mathrm{d}q^\varrho}{\mathrm{d}\tau} \frac{\mathrm{d}q^\sigma}{\mathrm{d}\tau} = 0 \quad . \tag{11}$$

4. – Consider a continuous "cloud of dust", described by a material energy-tensor $T^{\mu\nu} = \varrho\, v^\mu v^\nu$, where ϱ is the invariant mass density and v^μ the four-velocity of the "dust" elements. A thin spacetime tube of world lines represents the motion of a "dust" corpuscle. Now, as it is known, this motion follows a *geodesic* trajectory. Consequently, *no GW is emitted*. With a suitable choice of the reference frame, it is possible to prove that also the motions of the corpuscles of an electrically charged "dust" can be geodesically represented; nay, this result can be extended to the corpuscles of a "dust" which interact through any field of force [2].

5. – We read in Weyl [3] a penetrant analysis of a fundamental property of the general-relativistic coordinate systems. He wrote [3*b*)]: "... the concept of relative motion of several bodies has, as the postulate of general relativity shows, no more foundation than the concept of absolute motion of a single body. Let us imagine the four-dimensional world as a mass of plasticine traversed by individual fibers, the world lines of the material particles. Except for the condition that no two world lines intersect, their pattern may be arbitrarily given. The plasticine can then be continuously deformed so that not only one but all fibers become vertical straight lines." It is clear that this consideration implies that there exists always a coordinate transformation, which allows us to pass from any coordinate system for which some bodies are in motion to a *co-moving* coordinate system for which all these bodies are at rest.

Now, no class of privileged coordinate systems exists in GR, and any *physical* effect must be frame independent [4]. Consequently, the fact that bodies at rest cannot generate gravitational waves has a general significance: no coordinate system exists for which the motions of the bodies generate gravitational waves.

Remark that these considerations hold for the general case in which both gravitational and non-gravitational forces are present. For a different proof, founded on the Einstein field equations, that *all* the general-relativistic motions can be *geodesically* described, see our paper of ref. [2].

An immediate corollary: the gravitational field of a body, whose motion is geodesic, moves *en bloc* with the body, and is propagated *instantaneously*. In special relativity we have a partial analogue: the *static*-electromagnetic fields created by an electric charge in a rectilinear and uniform motion (a Minkowskian geodesic motion) move *along with* the charge, and are propagated *instantaneously* [5]. (This corresponds perfectly to the results found by an observer in a rectilinear and uniform motion, who travels with respect to the charge at rest).

This fact has been considered a paradox by some physicists, who have tried to get rid of it with the gratuitous surmise that it holds only for infinite spatio-temporal motions of the electric charge.

An instantaneous propagation of a field is not always in contradiction with the theory of relativity.

6. – The *wave-like nature* of undulatory metric tensors (with $R_{\iota\kappa\lambda\mu} \neq 0$, of course) *depends* on the reference system, *i.e.* is only a mathematical property of particular frames. Accordingly, these metric tensors do not represent physical GWs [6]. Remark further that the propagation velocity of any metric tensor depends on the reference system: with a suitable choice of general coordinates, this velocity can take any value between zero and the infinite.

We have here two precise consequences of the *general covariance* of GR.

7. – A famous thesis by Lorentz and Levi-Civita (which has been formally proved [7]) affirms that in Einstein field equations the material energy-tensor $T_{\mu\nu}$ is "balanced exactly" by $[R_{\mu\nu} - (1/2)g_{\mu\nu}\, R]\,/\kappa$, *which is the true gravitational energy-tensor*.

As Levi-Civita [8] emphasized, these facts have a momentous consequence: free waves and other purely gravitational phenomena are excluded. When the matter tensor $T_{\mu\nu}$ vanishes, the same must happen to the gravitational energy-tensor $[R_{\mu\nu} - (1/2)g_{\mu\nu}\, R]\,/\kappa$. "This fact entails a total absence of stresses, of energy flow, and also of a simple localization of energy." [8].

8. – In 1930 Levi-Civita [9] demonstrated that the functions $z(x)$, $[x \equiv (x^0, x^1, x^2, x^3)]$, of the characteristic hypersurfaces $z(x) = 0$ of Einstein field equations are solutions of the Hamiltonian equation

$$H := \frac{1}{2}\, g^{\lambda\mu}(x)\, \frac{\partial z(x)}{\partial x^\lambda}\, \frac{\partial z(x)}{\partial x^\mu} = 0 \quad . \tag{12}$$

According to Levi-Civita, the equation $z(x) = 0$ gives the law of motion of an *electromagnetic* wave-front. This interpretation is quite obvious when $g^{\lambda\mu}(x)$ has a *non*-undulatory form. If $g^{\lambda\mu}(x)$ has a wavy form, there is no reason to repudiate the above interpretation, because the undulatory nature of $g^{\lambda\mu}(x)$ *depends on the chosen system of coordinates*.

Remark that Levi-Civita's conception is the natural extension of that valid for the null lines of *spacial* relativity.

Thus, also the general relativity contains the basic law of *geometric optics – and independently of Maxwell equations*.

9. – Any electromagnetic ray is a null geodesic in any spacetime – as it is well known –, in particular, in the spacetime created by itself. Consequently, *no* undulatory, purely gravitational, and *autonomous* field is generated by the propagation of any electromagnetic wave in any spacetime manifold.

If the matter tensor $T_{\mu\nu}$ coincides with the electromagnetic energy-tensor $E_{\mu\nu}$, the Einstein field equations have as a necessary consequence that the equation

$$g^{\lambda\mu}\,\frac{\partial z(x)}{\partial x^{\lambda}}\,\frac{\partial z(x)}{\partial x^{\mu}}=0 \tag{13}$$

is the equation of the characteristics of both Einstein and Maxwell equations [10]. (For Maxwell equations see Whittaker [11]).

10. – The *inflationary models* [12] are based on the following postulates: *i*) the existence of GWs; *ii*) the existence of a "quantum gravity", *i.e.* of a quantized GR; *iii*) the existence of quantum vacuum fluctuations of the Einsteinian metrics.

These postulates are destitute of any physical reality, as we shall prove.

Ad i) The non-existence of GWs has been demonstrated *ad abundantiam* in the previous sections.

Ad ii) The coefficients $g_{\mu\nu}$, $(\mu,\nu = 0,1,2,3)$, of the Einsteinian metrics are *conceptually different* from the fields of any field theory, whose substrate is the fixed Minkowskian spacetime. The $g_{\mu\nu}$'s "are" the spacetime – a "plastic" spacetime –, they cannot be promoted to operator quantities. Therefore any reasonable program of a quantized GR is doomed to a failure. (See paper [13] for a detailed treatment of the question). The locution "quantum gravity" designs currently an inconsistent patchwork of disparate concepts and results of the quantum field theories acritically applied to general relativity.

Ad iii) the negation of statement *iii*) is an immediate corollary of *Ad ii*).

A final criticism: all the inflationary models lack of quantitative, mathematical treatments of the end of the action of the *inflation field* with the transition to Friedmann model.

11. – According to some authors (see, *e.g.*, [14]) an *indirect* experimental proof of the existence of GWs is given by the time decrease of the orbital period P_b of the binary pulsar B PSR1913+16. Since we have demonstrated, and in various ways, that in GR the GWs are non-existing physical objects, it is clear that the rate of change of P_b must have other causes, different from the emission of GWs, maybe the viscous losses of the unseen pulsar-companion, if it were, *e.g.*, a helium star. For a detailed treatment of the question, see paper [15].

A very good agreement between the measured value of $\mathrm{d}P_b/\mathrm{d}t$ and its theoretical evaluation has been obtained making use of the well-known quadrupole formula of the linearized approximation of GR – (for a standard derivation of this formula see [16]). Also post-Newtonian computations [17] give excellent agreement with the experimental results. However, these concordances are rather suspect, because both theoretical methods are misleading from the standpoint of the *exact* formulation of GR; in particular, they employ the unreliable notion of a gravitational *pseudo* energy-tensor (see Appendix, α)).

12. – The vain experimental researches for GWs by the gigantic LIGO-Virgo Collaboration have recently produced a paper entitled "Improved Upper Limits of the Stochastic Gravitational-Wave Background from 2009-2010 LIGO and Virgo Data" [18]. Stimulated by the (fictive) inflationary interpretation of the BICEP2 experiment [12], the LIGO-Virgo scientists aim at the discovery of the hypothesized stochastic GWs, that would be generated by superpositions of contributions by astrophysical and cosmological sources. However, they write in the abstract: "Consistent with predictions from most stochastic gravitational-wave background models, the data display no evidence of a stochastic gravitational-wave signal". And in the *Conclusions*: "With Advanced LIGO and Advanced Virgo detectors on the horizon, the sensitivity of interferometers to the SGWB will improve substantially in the coming years". Thus, they believe that their efforts will give a positive result. In reality, they are the victims of an *idée fixe*, and therefore they do not realize that the gravitational waves – like the phlogiston and the cosmic ether – belong to a past stage of the physical research.

APPENDIX – Some complementary remarks

α) *Pseudo* (*i.e.* false) gravitational momentum-energy-stress tensor: a spurious notion, which has been formulated in various ways, more or less "refined", and which has been unobjectionably rejected by Levi-Civita [8] with a stringent mathematical consideration. We have an object which is covariant only under *linear* coordinate-transformations, and that can be exactly reduced to zero at any point of the spacetime manifold. Further, we can create this pseudo tensor in a flat spacetime – *i.e.*, in the absence of gravitation – by simply employing curvilinear coordinates. Many authors have utilized it by assuming restrictions of its application field, giving origin to hybrid (and meaningless) formulae, containing generally-covariant terms and terms which are covariant only under linear coordinate-transformations.

β) At the ends of the Thirties of past century, Einstein, Rosen, Infeld and other relativists had lost the belief in the real existence of the GWs. Thus, in the following years various sceptical considerations on the GWs were developed. In particular, Infeld and his pupil Scheidegger were very active in this field; the Einstein-Infeld-Hoffmann method (see Infeld [1]) played a fundamental role. This method is a perturbative treatment of Einstein field equations: one develops all the functions φ's that appear in these equations into a power series of a small parameter λ:

$$(A1) \qquad \varphi(x,\lambda) = \sum_{l=0}^{\infty} \lambda^l \underset{l}{\varphi}(x) \quad ;$$

thus, in particular ($\mu, \nu = 0, 1, 2, 3$):

$$(A2) \qquad g_{\mu\nu}(x) = \eta_{\mu\nu} + \sum_{n=1}^{\infty} \lambda^n \underset{n}{h}_{\mu\nu}(x) \quad ,$$

if $\eta_{\mu\nu}$ is the usual Minkowski tensor.

To determine the motions of the point-masses of a discretized "dust" we have two perturbative approaches at our disposal: *i*) in the original EIH approach one searches the solutions of $R_{\mu\nu} = 0$, in perfect analogy with the mass-point solutions of Laplace equation $\triangle U = 0$; *ii*) in Infeld's approach one follows the procedure of sect. **2**. (Remark that the employment of Dirac's delta-functions is analogous to their employment in Newton theory).

In the approximations higher than the second there are terms describing a gravitational-radiation damping. However, one can perform *at any stage* a suitable coordinate transformation which reduces them to zero: the equations of motion acquire a "Newton-like" form [19] . This result is conceptually fundamental: it gives a significant corroboration of our *exact* result of sect. **2**. The objection that Infeld has not postulated an "Ausstrahlungs-bedingung" is meaningless: in GR we can perform any continuous coordinate transformation.

γ) *Linearized approximation – or linear version – of GR.* There are two methods of deduction, the usual one [20] and that of Weyl [21]. In the first, one starts from the approximate equality $g_{\mu\nu} \approx \eta_{\mu\nu} + \varepsilon h_{\mu\nu}$, where ε is a small parameter, and one remarks that the symmetric field $h_{\mu\nu}$ is covariant *only* under the Lorentz transformations of the coordinates. Weyl [21] deduces the linear version of GR quite *independently* of the Einstein field equations. He proves that there is a linear field theory of gravitation in a Minkowskian spacetime, which has the gauge-invariance property

$$(A3) \qquad h^{*}_{\mu\nu} = h_{\mu\nu} + \frac{\partial \lambda_\mu}{\partial x^\nu} + \frac{\partial \lambda_\nu}{\partial x^\mu} \quad ,$$

where the four functions χ_μ are arbitrary, that gives the same equations of the conventional procedure [20]. However, in his paper [21] Weyl adds a conceptually fundamental remark: the trajectories of the particles of a "cloud of dust" are Minkowskian geodetics, *i.e.* their four-velocities u^μ satisfy the equations $\mathrm{d}u^\mu/\mathrm{d}s = 0$: rectilinear and uniform motions. "From the standpoint of Einstein's theory this is as it should be, because *the gravitational force arises only when one continues the approximation beyond the linear stage*." (It is clear that this conclusion does not concern the computations of the geodesics of a *test*-particle, or of a light-ray, in a *given* approximate field $g_{\mu\nu} \approx \eta_{\mu\nu} + \varepsilon h_{\mu\nu}$).

Of course, the field $h_{\mu\nu}$ can be quantized, but by virtue of the classical equations $\mathrm{d}u^\mu/\mathrm{d}s = 0$ *no* GW is emitted – the *graviton* is a science-fiction object.

REFERENCES

[1] L. Infeld, *Acta Phys. Polonica*, **13** (1954) 187 – and the references therein. See also L. Infeld and J. Plebanski, *Motion and relativity* (Pergamon Press, Oxford, *etc.*) 1960.

[2] A. Loinger and T. Marsico, *arXiv:1066.3844* [physics.gen-ph] 19 Jun 2010.

[3] *a*) H. Weyl, *Raum-Zeit-Materie*, Siebente Auflage (Springer-Verlag, Berlin, *etc.*) 1988, p.268, *b*)*Idem*, *Philosophy of Mathematics and Natural Science* (Princeton University Press, Princeton) 1949, p.105; *c*) *Idem*, *Mathematische Analyse des Raumproblems* (J. Springer, Berlin) 1923, Siebente Vorlesung; an Italian translation by A. Loinger: *Analisi matematica del problema dello spazio* (Zanichelli,

Bologna) 1991, Settima lezione.

[4] D. Hilbert, *Gött Nachr.*: Erste Mitteilung (20. Nov. 1915); zweite Mitteilung (23. Dez. 1916) — *Math. Annalen*, **92** (1924) 1; also in *Gesammelte Abhandlungen*, Dritter Band (J. Springer, Berlin) 1935, p.258.

[5] See, *e.g.*, R. Becker and F. Sauter, *Electromagnetic Fields and Interactions*, vol. I (Blaisdell Publ. Company, New York, *etc.*) 1964, sect. **69**; J. D. Jackson, *Classical Electrodynamics* (J. Wiley and Sons, Inc., New York, *etc.*) 1967, sect. **14.1**. See further A. Loinger, *arXiv:physics/0506024*, June 4th, 2005.

[6] A. Loinger, *arXiv:gr-qc/9909091*, Sept. 30th, 1999.

[7] A. Loinger, *arXiv:1109.5268* [physics.gen-ph], 24 Sep 2011.

[8] T. Levi-Civita, *Rend. Lincei*, **26** (1917) 381; see *arXiv:physics/9906004* for an English translation , June 2nd, 1999.

[9] T. Levi-Civita, *Rend. Acc. Lincei*, s.6a, Vol.XI (1930), 3; *ibidem*, 113.

[10] A. Loinger and T. Marsico, *arXiv:1106.2210* [physics.gen-ph] 11 Jun 2011.

[11] E.T. Whittaker, *Proc. Cambridge Phil. Soc.*, *XXIV/1* (1927) 32.

[12] See, *e.g.*, the references in BICEP2 *Coll.*, *PRL* **112**, 241101 (2014).

[13] A. Loinger, *arXiv:physics/0308042* — August 12th, 2003.

[14] J.M. Weisberg and J.H. Taylor, *Binary Radio Pulsars*, *ASP Conference Series*, *Vol. TDB*, 2004. Also in *arXiv:astro-ph/0407149* — July 7th, 2004.

[15] A. Loinger, *arXiv:physics/0502089* — February 16th, 2005.

[16] L. Landau et E. Lifchitz, *Théorie du Champ* (Éditions MIR, Moscou) 1966, sect. **104**.

[17] For a standard treatment of the methods, see, *e.g.*, L. Blanchet, *arXiv:0907.3596* [gr-qc] — March 11th, 2010.

[18] LIGO-Virgo *Coll.*, *arXiv:1406.4556* [gr-qc] August 12th, 2014.

[19] A. Scheidegger, *Revs. Mod. Phys.*, **25** (1953) 451.

[20] See, *e.g.*, W. Pauli, *Teoria della Relatività* (Boringhieri ed., Torino) 1958, sect. **60**.

[21] H. Weyl, *Amer. J. Math.*, **66** (1944) 591.