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# RELATIVISTIC GALAXY ROTATIONS *VERSUS* DARK MATTER

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SUNTO. — Gli effetti gravitazionali attribuiti alla materia oscura possono essere spiegati per tutti i tipi di galassie con un modello di relatività generale delle rotazioni galattiche.

ABSTRACT. — The gravitational effects ascribed to the Dark Matter can be explained with a general-relativistic treatment of the galaxy rotations.

SUMMARY. — **1**. Aim of the paper. — **2**. Core of the question and results from 1937 to the present time. — **3**. A résumé of the general-relativistic approach developed in a previous paper, which holds for all kinds of galaxies. — **4**. Application to elliptical N3379 and comparison with the observational data. — **5**. On the lensing phenomena. — **6**. Conclusion. — *Appendix*.

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1. — We have proved in a previous paper ("A relativistic model of the galactic rotation") [1] the statement of the present abstract for spiral galaxies, considering in particular the instance of the Milky Way.

We shall now demonstrate that our approach holds for all types of galaxies, *e.g.* for the ellipticals.

2. — The pioneering memoir by Zwicky (1937) [2] and the recent observational studies of the galaxies [3] emphasize that their fundamental properties are essentially very similar. In the ensemble of stars, which are near to the supermassive centre of a galaxy, the stellar circular velocities increase linearly with the distance from the centre, as in the rotations of

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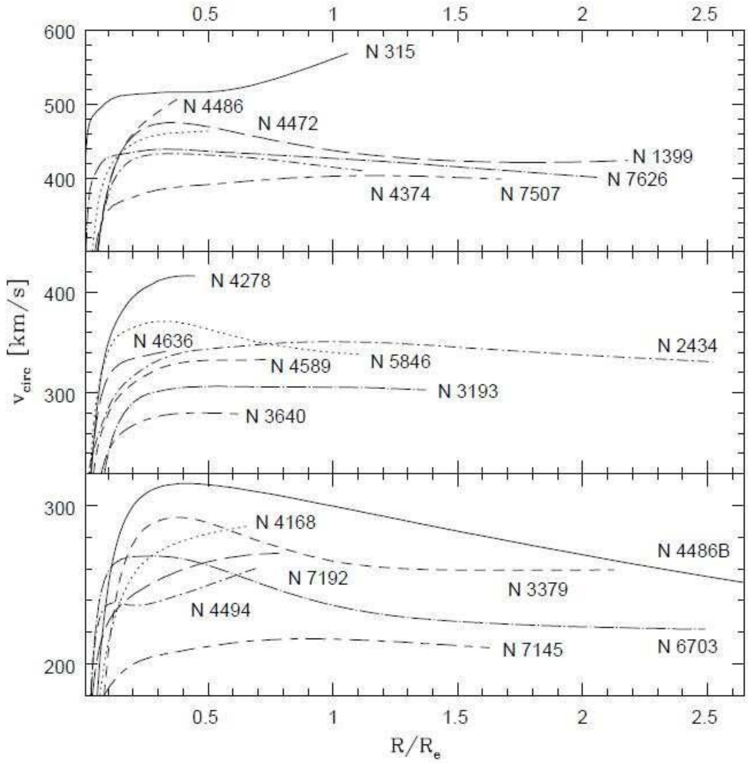


*Fig.* **1**. [*Fig.* **1** *of Gerhard [3]*]. *The "best model" circular velocity curves of all galaxies from the* $K + 2000$ *sample plotted as a function of radius scaled by the effective radius* $R_e$. *The panels are roughly ordered by luminosity.*

a rigid body. In the ensemble of the far stars, the velocities decrease very slightly and tend to become constant.

According to Zwicky (1937) [2], the behaviour of the stars of the first ensemble has its origin in a sort of (gravitational !) "viscosity" among them. The present researchers [3] think that it is originated by a "conspiracy" between the luminous matter and a fraction of dark matter. The stellar motions in the second ensemble are dominated by the gravitational action of a dark matter halo – according to a widespread interpretation.

See in *Fig.* 1 the observational diagrams of the circular-velocity curves (CVCs) of stars in some elliptical galaxies [4]. (To describe the star orbits with circular trajectories represents a reasonable *model*).

3. – Let us summarize our mathematical treatment of sect. **2** in paper [1].

We start from the well-known formula – in spherical polar coordinates $r, \vartheta, \varphi$ – of Schwarzschild's manifold created by a concentrated mass $M$ at rest. We assume a general representation $\mathcal{R}(r)$ for the radial coordinate, with $\mathcal{R}$ a regular function of $r$. We put $m \equiv GM/c^2$ and $\alpha \equiv 2m$. We are interested in the *circular* geodesics ($\mathrm{d}\mathcal{R} = 0$) of the test-particles (which represent the stars in our model) and of the light-rays, moving in a plane, *e.g.* in the plane $\vartheta = \pi/2$.

Let $\Omega > 0$ be the angular velocity of rotation of the supermassive centre of a given galaxy, which we represent by the Schwarzschildian mass $M$. The geodesics will be modified by this rotation.

The substitution $\varphi \to \varphi - \Omega t$ in Schwarschild's formula gives the changed metric for the stars of the first ensemble. It is not difficult to see (*vide* [1]) that their circular velocity $v_1$ is:

(1) $$v_1 = (\mathcal{R}\dot{\varphi})_1 = \mathcal{R}\,\Omega + c\sqrt{\frac{\alpha}{2\mathcal{R}}} \quad ,$$

where $\dot{\varphi} \equiv \mathrm{d}\varphi/\mathrm{d}t$. Eq. ((1)) holds from $\mathcal{R} = (3/2)\alpha$ to a certain value, say $\tilde{\mathcal{R}}$, observationally determined, of $\mathcal{R}$. (We leave out for brevity the treatment of the light-rays).

The function $v_1(\mathcal{R})$ has a minimum at the following value $\mathcal{R}_{\mathrm{m}}$ of $\mathcal{R}$:

(2) $$\mathcal{R}_{\mathrm{m}} = \frac{1}{2}\left(\frac{\alpha\, c^2}{\Omega^2}\right)^{1/3} \quad ; \quad \Rightarrow$$

(3) $$\dot{\varphi}_1(\mathcal{R}_{\mathrm{m}}) = 3\Omega \quad .–$$

For the stars of the second ensemble – from $\mathcal{R} = \tilde{\mathcal{R}}$ to a given observational value $\mathcal{R} = \mathcal{R}_{\mathrm{f}}$, we put, where $u$ is a positive constant:

(4) $$\varphi \to \varphi - \frac{u}{R}\,t \quad ; \quad \Rightarrow$$

(4′) $$\dot{\varphi} \to \dot{\varphi} - \frac{u}{R} \quad .$$

The agreement of the relativistic metrics of the two ensembles at $\mathcal{R} = \tilde{\mathcal{R}}$ gives

(5) $$u = \tilde{\mathcal{R}}\,\Omega \quad ,$$

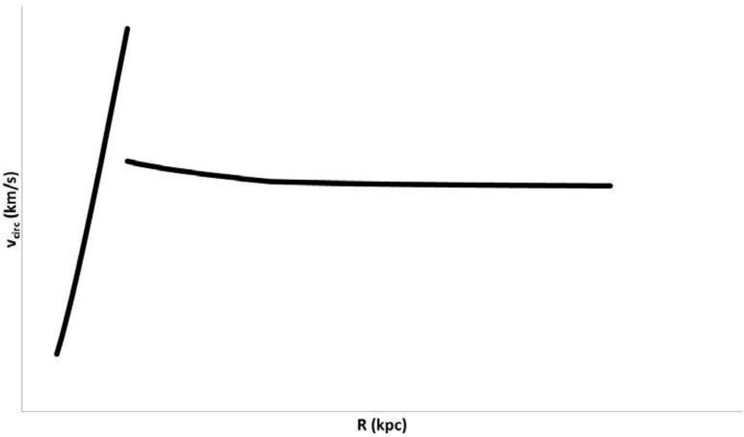


*Fig.* **2**. *Our theoretical CVC form, which holds for all the galaxy types (spirals, ellipticals, lenticulars, etc.). Of course, for numerical parameters in the observational range.*

and the velocity $v_2$ of the stars of this ensemble is:

$$v_2 \equiv (\mathcal{R}\dot{\varphi})_2 = \frac{\tilde{\mathcal{R}}\Omega}{2} + \sqrt{\frac{\tilde{\mathcal{R}}^2\Omega^2}{4} + \frac{\alpha c^2}{2\mathcal{R}}} \quad . \tag{6}$$

A qualitative diagram of the form of the functions $v_1$ (eq.((1))) and $v_2$ (eq.((6))) is given in *Fig.* **2**. It says that for *all* galaxy types of Hubble's classification the Einstein field equations describe perfectly the gravitational effects ascribed to the Dark Matter.

*N.B.* – As in [1] we assume that $\mathcal{R}(r)$ represents also the distance from the centre of the considered galaxy (see [5]). –

The theoretical diagram of *Fig.* **3** regards the circular-velocity curve of galaxy N3379. In *Fig.* **1** we see its CVC as given by an observational model.

The pertinent numerical parameters are given in the following sect.4.

4. – The abscissae of the CVCs in *Fig.* **1** are $R/R_e$, where $R$ is the radius of a circular orbit and $R_e$ gives the effective radius of the considered galaxy. For galaxy N3379 (see the bottom of *Fig.* **1**) we have $R_e = 2.236$ kpc $= 2.236 \times 3.087 \times 10^{16}$ km. (We recall that $R_e$ is the radius of a circumference

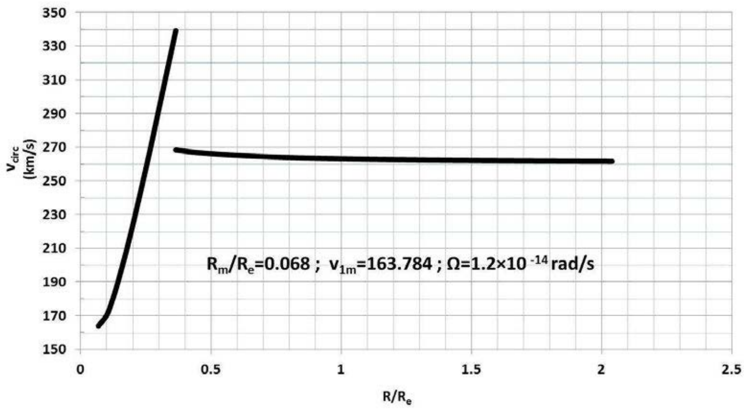


*Fig.* **3**. *Our theoretical diagram of the CVC of N3379.*

that encloses a region which emits a luminous power equal to the half of the power emitted by the galaxy).

The mass $M$ of the supermassive centre of N3379 is $4.16 \times 10^8$ solar masses; therefore

$$\alpha = 1.248 \times 10^9\,\mathrm{km} \quad . \tag{7}$$

$\tilde{\mathcal{R}}$ is the abscissa of the maximal circular velocity (see *Fig.* **1**); we have:

$$\tilde{\mathcal{R}} = 0.365 \times \mathcal{R}_e \quad . \tag{8}$$

The angular velocity $\Omega$ of the supermassive centre is such that $\tilde{\mathcal{R}}\Omega$ is equal, with a good approximation (cf. eq.((6))), to the circular velocity of the last star. We obtain:

$$\Omega = 1.2 \times 10^{-14}\,\mathrm{rad/s} \quad . \tag{9}$$

With these parameters we compute $v_1 = (\mathcal{R}\dot{\varphi})_1$ and $v_2 = (\mathcal{R}\dot{\varphi})_2$ – and the minimum $v_1(\mathcal{R}_\mathrm{m})$ of $v_1(\mathcal{R})$:

$$v_1(\mathcal{R}_\mathrm{m}) = v_1\left[\frac{1}{2}\left(\frac{\alpha c^2}{\Omega^2}\right)^{1/3}\right] = \frac{1}{2}\left(\alpha\, c^2\,\Omega\right) + c\sqrt{\alpha}\left(\frac{\alpha\, c^2}{\Omega^2}\right)^{-1/6} \quad . \tag{10}$$

The results are graphically represented in *Fig.* **3**.

5. – Since our theoretical model yields the same gravitational effects of

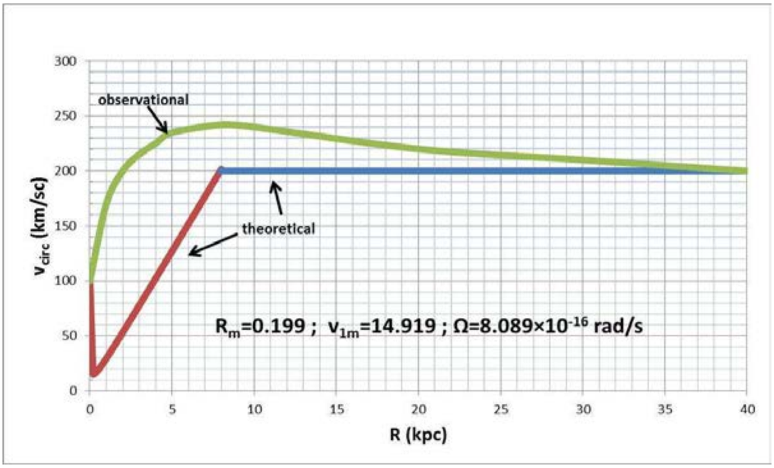


*Fig.* **4.** *The theoretical CVC of the Milky Way for* $\Omega = 8.089 \times 10^{-16}$ *rad/s. It differs very little from that of the paper [1].*

the Dark matter, it is obvious that it describes rightly all *lensing* observational data.

6. – We emphasize finally that our model rests on the Einsteinian field equations of general relativity without the Λ-term [6] and *without any modification*.

It is a remarkable fact that Zwicky (see pp. 220÷ 223 of [2]), starting from a non-relativistic and intuitive viewpoint, wrote some formulae that resemble our relativistic results.

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## *APPENDIX*

If we determine the Ω of the Milky Way with the criterion adopted in sect.**4** for N3379, we get $\Omega = 8.0 \times 10^{-16}$ rad/s. Consequently, we have a new CVC, see *Fig.* **4**.

[The theoretical diagram of *Fig.* **2** in paper [1], for which $\Omega = 8.855 \times 10^{-16}$ rad/s, does not show that $v_1(\tilde{\mathcal{R}}) > v_2(\tilde{\mathcal{R}})$; however, the difference $v_1(\bar{\mathcal{R}}) - v_2(\bar{\mathcal{R}})$ is small. *Idem* for the CVC of *Fig.* 4 of the present paper.]

A last remark. The $\Omega$ of the previous paper [1] and the $\Omega$ of the *Fig.* 4 differ very little ($8.855 \times 10^{-16}$ and $8.089 \times 10^{-16}$ rad/s). This fact is interesting. The first $\Omega$ is the angular velocity of the revolution orbit of the Sun (a peripheral star), as it follows from the *Newtonian* attraction between the Sun and the total mass of the Milky Way ($\approx 10^{11}$ solar masses.)

The second $\Omega$ is the angular velocity of the supermassive centre of our galaxy ($4.5 \times 10^6$ solar masses), as it follows from the *Einsteinian* rotation and the observational circular velocity of the galactic last star.

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[4] See *Fig.* **1** in Gerhard [3].

[5] Cf. App. B in A. Loinger and T. Marsico, *arXiv:1011.2600* [physics.gen-ph] 11 November 2010.

[6] We have employed GR without the $\Lambda$-term also in our paper "The Dark Energy explained by a Schwarzschildian cosmology — *et cetera*", *Academia.edu* and *Researchgate*, 2014 March.