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THE COLOSSEUM
Dynamic Behaviour and Monitoring of the Monument

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SUNTO. – Il Colosseo aveva in origine una struttura molto robusta con una forma chiusa di anelli concentrici e pareti radiali collaboranti. In seguito a cedimenti della fondazione e agli effetti di terremoti la parte esterna sul lato sud è crollata e la struttura è divenuta più vulnerabile. Per mantenere questa situazione è necessaria un'adeguata attività di monitoraggio. In questo contesto un ruolo importante assume il monitoraggio dinamico basato sull'osservazione nel tempo di alcune proprietà dinamiche della struttura. Dopo un cenno di dinamica strutturale, si evidenzia che le grandezze modali, frequenze e modi di vibrazione, sono adatte al monitoraggio perché sono semplici da determinare sperimentalmente e variano a seguito di un danno. La tecnica è mostrata considerando due indagini sperimentali che hanno interessato il Colosseo nel 2006 e 2022. Inoltre, sulla base di registrazioni delle accelerazioni prodotte dal traffico e da un recente terremoto, si affronta il problema delle vibrazioni dovute al traffico e si sviluppano considerazioni preliminari sulla vulnerabilità sismica del monumento.

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ABSTRACT. – The original design of the Colosseum was a robust, closed structure with concentric rings and radial walls. However, due to foundation settlements and earthquake vibrations, a significant portion of the external zone on the south side has collapsed, making the structure more vulnerable. To maintain the current situation and prevent further collapses, it is essential to support monitoring activities focused on the structural health conditions of the monument. This study centers on dynamic monitoring based on variations in specific dynamic properties over time. After reviewing some elements of structural dynamics, modal quantities, frequencies and vibration modes, are

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shown to be suitable for monitoring because they can be easily determined experimentally and are sensitive to damage. The technique has been applied comparing the results obtained in two experimental campaigns in 2006 and 2022, where the Colosseum response to ambient excitation has been measured. Additionally, recorded accelerations from road traffic and a recent earthquake enable an assessment of traffic vibrations and a preliminary evaluation of the seismic vulnerability.

1. Introduction

The Colosseum is the most famous monument of ancient Rome and, as such, receives special attention from the Superintendent of Cultural Heritage, with a variety of conservation activities. Currently, a new underground metro line that crosses the center of Rome is under construction, running tangentially to the Colosseum [1]. This project has provided an excellent opportunity to carry out an extensive experimental investigation campaign on the monument material properties and dynamic characteristics during the design phase. Furthermore, the Colosseum has been equipped with a permanent sensor network to enable both static and dynamic monitoring.

In recent decades, Structural Health Monitoring (SHM) has gained significant importance. The increasing prevalence of SHM is partly due to advancements in instrumentation and reductions in associated costs. Concurrently, data processing methods and analytical techniques have become more sophisticated and effective. In this context, vibration-based methods for monitoring structural health have become well-established, particularly because valuable information can be gathered efficiently using ambient excitations, which have proven effective in detecting structural deterioration. For large structures and cultural heritage sites like the Colosseum, SHM should be considered essential [2,9].

In this paper, after providing general information on the history of the monument and a description of its current state, the focus shifts to the fundamentals of structural dynamics, which are essential for understanding the health monitoring approach based on the evolution of dynamic properties of the structure. The paper then examines the response of the monument to two significant dynamic excitations, traffic and earthquakes. This analysis is supported by available data from the monitoring instruments currently in operation.

2. The History of the Monuments and its Structure

The Flavian Amphitheater, more commonly known as the Colosseum, was constructed between 70 and 80 AD under the emperors Vespasian and Titus of the Flavian dynasty. The foundations rest on heterogeneous soil in an area that was once partially occupied by the artificial lake of the Domus Aurea. The Colosseum hosted gladiatorial games until 523 AD, when Emperor Theodoric banned them. Afterward, the structure entered a long period of decline until its restoration in the 19th century.

The amphitheater is elliptical in shape, with the major and minor axes measuring 188 meters and 156 meters, respectively. The structural design follows the well-established Roman technique for masonry buildings with orthogonal walls, adapted here to the elliptical form. A series of radial masonry walls, tied by concentric annular walls and inclined concrete vaults, creates concentric wedge-shaped cells (*Fig. 1*).

The materials used include travertine, tuff, masonry, and concrete. The external wall, standing approximately 50 meters high, was originally made of travertine blocks connected by iron pins and cramps without mortar joints, many of which have disappeared over the centuries. The engineers of the time were aware of the varying deformability of the soil across the large area occupied by the monument, and a large and deep foundation was built as a concrete ring, twelve meters thick. However, the northern side of the Colosseum is built on stiffer soil than the southern side (*Fig. 2a*).

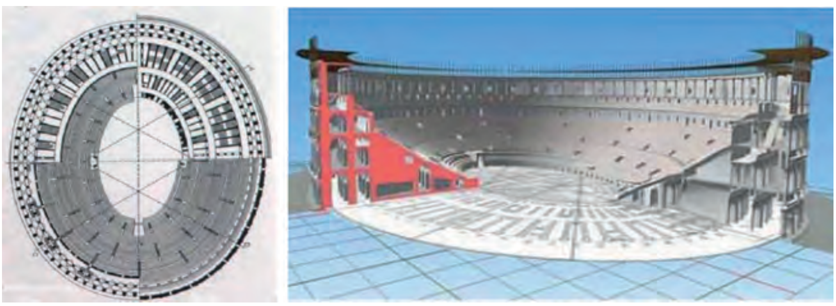

Fig. 1. Plan and sectional view of a model: the original design of the monument, characterized by its closed elliptical shape, composed of interconnected radial and circumferential walls, symmetrical with respect to the principal axes.

This geometrical configuration originally provided the structure with significant robustness against both vertical and horizontal forces, such as those induced by earthquakes. Unfortunately, these exceptional characteristics were compromised due to the varying deformability of the soil on which the foundation rests. Despite the rigidity of the base concrete ring, settlements on the south side caused discontinuities in the elevated structures. In the absence of internal circumferential constraints and the lack of iron pins between the blocks, the structure was unable to resist seismic horizontal forces. The partial collapse of the external walls led to the current world-renowned profile of the Colosseum (*Fig. 2b*), where the original symmetry has been partially lost, resulting in a negative impact on the structural robustness of the monument, as will be discussed later.

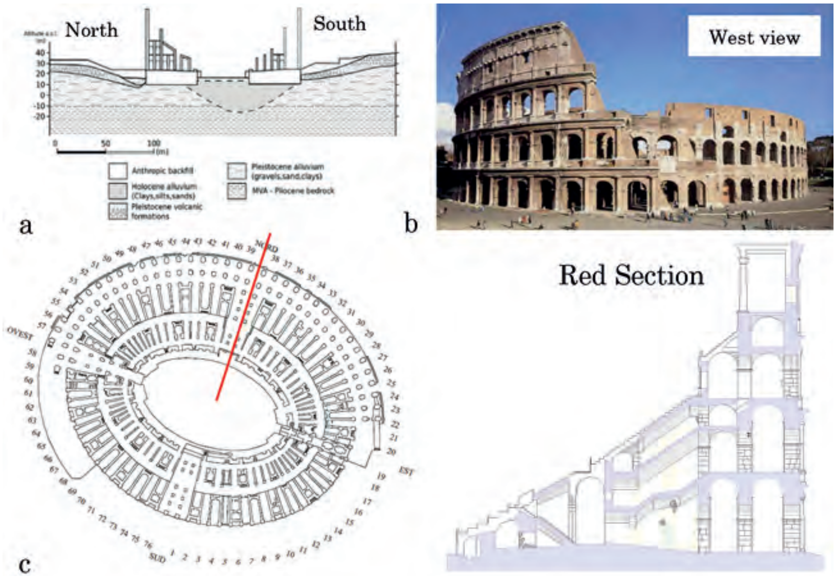

Fig. 2. a) A transversal section of the monument with the different soils under the base foundation; b) the iconic image of the Colosseum; c) actual plane and section of the North side.

In the 19th century, several interventions were carried out to halt the degradation of the monument. The most notable are the two large buttresses constructed at the eastern end by Stern (1805–1807) and at the western end by Valadier (1820–1852), as shown in *Fig. 2b*. These buttresses were built to support the remaining portion of the external wall, attempting to compensate for the missing southern sections.

Despite these restoration efforts and the absence of recent severe earthquakes, various decay phenomena have occurred over time. Currently, the health status of the Colosseum is not very satisfactory, and the effectiveness of the connections between the bearing elements remains uncertain. An important factor to consider is that Rome is classified as a seismic zone, albeit with low intensity. For all these reasons, it is evident that monitoring of the monument is needed to assess its health status and to record potential vibrational events.

3. The Structural Dynamic Monitoring

Structural health monitoring (SHM) has garnered increasing attention in recent times. This interest arises from the need to assess the integrity of constructions and to plan maintenance interventions. The principles underlying monitoring are based on observing variations in certain structural characteristics. Monitoring methods differ according to the quantities observed, resulting in a wide variety of techniques developed [3,9]. A critical aspect of selecting the observed quantities is that they must be easily measurable and primarily related to the evolution of damage.

In this context, the dynamic properties of a structure meet these requirements satisfactorily. Indeed, they are distinct quantities of the structure, related to its health conditions, and can be experimentally determined in a simple manner. Before discussing the dynamic monitoring of a structure, it is necessary to introduce the fundamental descriptors of dynamic behaviour and their variations due to damage.

3.1 Dynamics of a structure

The study of the response of a mechanical system requires the use of a model. When the structure is relatively simple, such as the frame structure shown in *Fig. 3* with a rigid horizontal beam, it can be approximated by the simplest model: a single-degree-of-freedom (SDOF) system. In this model, the deformed shape is described by a single variable $u_1(t)$, the top mass displacement.

Fig. 3. The structure (a) and its deformed configuration (b) of a simple model described by only one variable $u_1(t)$*, time history of the free oscillations (c).*

When this structure is displaced from its initial position and then released, it oscillates in harmonic motion, characterized by a period, which is the time interval between two successive maxima (*Fig. 3c*). The period is a distinctive property of the structure and depends on both its mass and stiffness. Alternatively, it may be useful to refer to the frequency of the structure, which is the inverse of the period. The frequency indicates how many times $u_1(t)$ reaches its maximum within a unit of time.

It is worth noting that this quantity, the period or the frequency, depends directly on the stiffness of the structure, and thus indirectly reflects its structural health condition. For example, if a column in the frame is damaged, its stiffness decreases, causing the oscillation period to increase (or the frequency to decrease). In this scenario, when the same experiment shown in *Fig. 3* is repeated with a damaged column, the frame response is slower (*Fig. 4*). The damage produces a noticeable effect, making the oscillations visibly different.

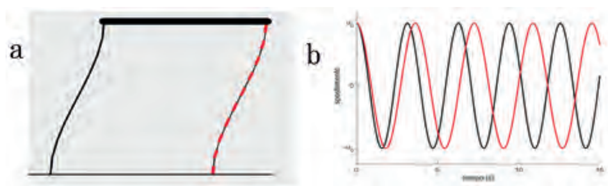

Fig. 4. The structure with a damaged column (red); (b) time histories of free oscillations of undamaged (black) and damaged (red) structures.

It can be concluded that the period (or frequency) of a structure is a crucial parameter to monitor over time. Its variation provides valuable information regarding the health of the structure, making it a useful indicator in structural health monitoring [6-9].

The structures we are discussing, such as the Colosseum, are obviously much more complex than the simple frame of *Fig. 3*. To capture their dynamic behaviour, we need to consider systems with more than one degree of freedom. A representative case of a multi-degree-of-freedom (MDOF) structure is shown in *Fig. 5*, where the deformed shape of the structure is described by three variables $u_i(t)$, with i=1,3, representing the displacements of the three masses. While this example remains relatively simple, it can illustrate the dynamic properties of a system with multi-degrees of freedom.

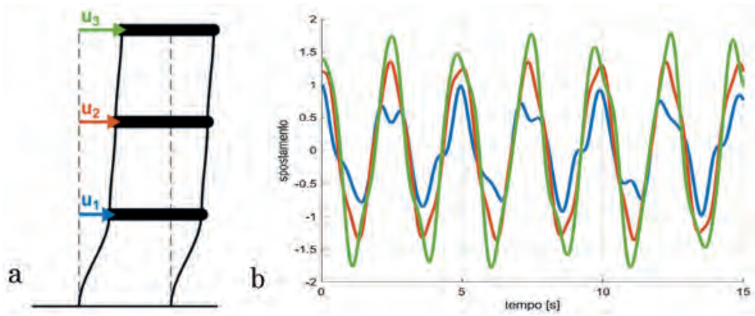

Fig. 5. A 3DOF structure (a) and the time hystories of the three displacements $u_1(t)$, $u_2(t)$, $u_3(t)$ *during a free oscillation caused by a generic initial condition (b).*

If the structure is displaced from its undeformed configuration, much like the SDOF system in *Fig. 3*, its response is now described by the time histories of the 3DOF system (*Fig. 5*). However, unlike the SDOF system, the response is far from harmonic. In this case, the initial deformed configuration is arbitrarily assumed, but there are countless possibilities - potentially ∞^3, given the three degrees of freedom - to define the shape of the initial deformation.

In this case, the mathematical analysis of the dynamic equations of motion for the model, which analytically represent the motion of the structure, provides an important result: for this model with three degrees of freedom, there exist three specific initial deformed configurations that generate free oscillations characterized by harmonic motion with periods T_1, T_2, T_3, where $T_1<T_2<T_3$. In these oscillations, the deformed shape remains constant over time while only its amplitude changing.

These deformed shapes, known as natural modes, or simply modes, are illustrated in *Fig. 6*. If the system is displaced from its undeformed configuration according to the *i-th* mode and left to oscillate freely, it will oscillate harmonically with a period T_i, or frequency f_i, while maintaining the same shape throughout the motion. Since the shape is known and does not change, the system motion can be described by only one variable, the displacement of one mass, meaning it behaves like a single-degree-of-freedom system. In a sense, the 3DOF structure can be viewed as comprising three SDOF systems, each characterized by a period, a specific mode shape and a variable representing its amplitude.

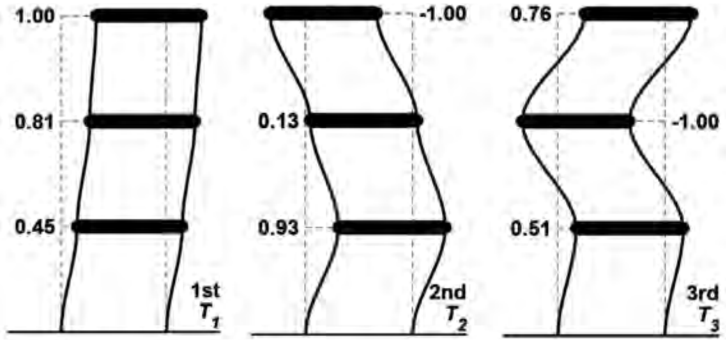

Fig. 6. The three modes of the structure: numerical values of masses displacements and deformed shapes.

3.2 Damage identification

Returning to the definition of the quantities that characterize the dynamic behavior of a multidegree-of-freedom (MDOF) structure, we can now list the periods (or frequencies) and the associated modes for each period. As previously mentioned, periods or frequencies are sensitive to damage, but modes are also sensitive to damage.

In particular, while frequency variations, being global quantities, cannot directly indicate the location of the damage, variations in the mode shapes - and especially in their derivatives, such as modal curvatures - are more sensitive to the localization of the damage [9]. Essentially, both periods and modes are reliable indicators for monitoring the evolution of a structure health. They are relatively easy to measure, and they show variations due to damage. However, while these variations can help in detecting the occurrence of damage, determining the precise location and extent of the damage involves solving an inverse problem, which is complex and does not always yield a unique solution. Achieving reliable results requires a robust interpretative model and accurate experimental data [9].

At this stage, monitoring of the monument can reliably depend on measuring modal quantities, such as frequencies and certain components of mode shapes. By comparing the variations in the experimental results between undamaged and damaged conditions with the corresponding values provided by the structural model (which depend on the stiffness properties), it is possible to determine these parameters, which are directly related to the extent of the damage.

Roughly speaking, if $\mathbf{z}$ is a vector representing the observed modal quantities, the relationship between the experimentally measured modal quantities $\mathbf{z_e}$ and the numerically computed modal quantities $\mathbf{z(x)}$, which depend on the mechanical parameters $\mathbf{x}$, directly related to damage, can be described as:

$$z_e - z(x) = e \tag{1}$$

where e accounts for the unavoidable discrepancies between the experimental and numerical quantities due to the presence of experimental and modeling errors. The solution of the inverse problem - given $\mathbf{z_e}$, determine $\mathbf{x}$ - can be pursued by minimizing a suitable objective function of the difference e between the experimental and numerical values. The variation over time of the parameters $\mathbf{x}$ provide valuable information about the portion of the structure affected by damage and its severity [7-9].

As is well known, solving this inverse problem is significantly more challenging than the direct problem, where the model is already established, and the response to a known external excitation is determined. In contrast, in the inverse problem the response (derived from experimental data) is known, and the objective is to determine the model parameters, which are related to damage. Achieving a reliable solution requires highly refined models and good experimental data, as previously noted.

Nevertheless, if no significant variation in the modal quantities is observed across several successive tests, it can be confidently concluded that no global degradation in the structural characteristics has occurred. This outcome of the ongoing monitoring would provide assurance that the monument overall structural health remains intact.

4. The Case-Study of the Colosseum

To determine the structural behavior of the monument and its modal quantities, the finite element method (FEM) is employed. In this approach, the structure is divided into a large number of small solid elements, ensuring an accurate representation of the monument geometrical and mechanical characteristics (*Fig. 7*). Furthermore, this technique allows for the selection of appropriate materials for each section of the structure, considering the specific mechanical and inertial properties of the various materials used (*Fig. 8*). In this particular case, four different materials are considered: travertine, tuff, masonry, and concrete.

For monitoring purposes, the Colosseum has been equipped with accelerometers to measure its vibrations. Two verticals, B and C, are continuously monitored by fixed sensors (*Fig. 9*). Additionally, other verticals are monitored using six accelerometers at a time, allowing for a broader collection of data across different parts of the monument.

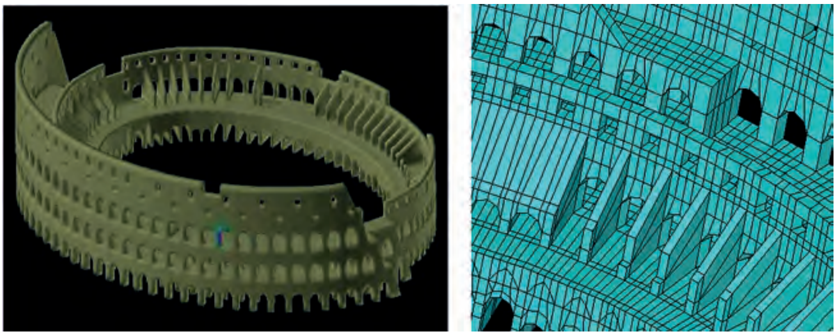

Fig. 7. The finite element model of Colosseum with a zoom of small zone.

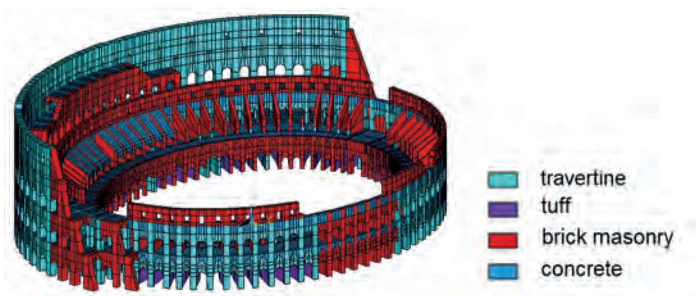

Fig. 8. Description of the zones with different materials.

Referring to the results of an experimental investigation conducted in 2006 [10], the model was used to carry out the modal analysis. The calculated frequencies are presented in *Tab. 1* and they were derived by assuming standard elastic material properties, which are well established in the literature [10]. However, a significant discrepancy was observed - the ratio f_a/f_e is around 2 - between the experimental f_e and analytical f_a frequencies. This discrepancy suggests that the elastic stiffness of materials might be overestimated in the model. This is likely due to the fact that the actual stiffness values are not specific to the materials of individual elements themselves, but rather to their assemblies. For instance, many of the vertical structural elements consist of assemblies of blocks, and the connections between these blocks are uncertain, likely relying on only a few contact points. This introduces a degree of flexibility that was not accounted for in the initial stiffness assumptions.

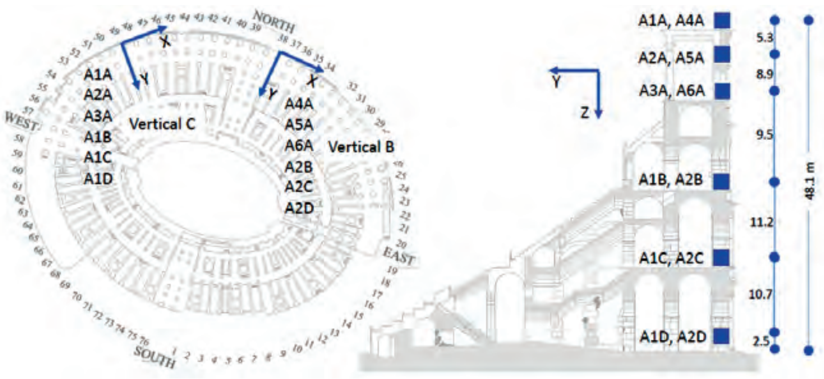

Fig. 9. Two instrumented verticals, B and C, with fixed positions of the accelerometers and sensors positive directions, plan and section.

Tab. 1. Experimental and numerical values (Hz) of the first six frequencies.

Mode	1	2	3	4	5	6
Experimental	**1.03**	**1.30**	**1.49**	**1.60**	**1.66**	**1.75**
Analytical original	2.32	2.34	3.12	3.17	3.58	3.61
f_a/f_e	2.21	1.80	2.09	1.98	2.16	2.07
Analytical updated	**1.13**	**1.14**	**1.51**	**1.54**	**1.74**	**1.76**

To further investigate this discrepancy and obtain a more accurate estimation of the mechanical properties representing these global structural elements, wave propagation phenomena are used [10]. There is indeed a relationship between the propagation velocity of pressure waves c_p and the continuum elastic properties:

$$c_p = \sqrt{\frac{E}{\rho}\frac{1-\nu}{(1+\nu)(1-2\nu)}} \tag{2}$$

where: E is the Young's modulus, ν is Poisson's ratio and ρ is the material density. This relationship allows for a more precise estimation of the effective stiffness, which considers the actual conditions of the structural assemblies rather than single material properties. The mean experimental value obtained for the pressure wave propagation speed c_p is 1220 m/s. This value leads to an estimate of the elastic modulus E that is significantly lower than the originally assumed value. Based on this experimental result, the elastic modules have been suitably scaled by a factor. The optimal scaling factor was determined by minimizing the differences between the experimental and numerical frequencies calculated by the finite element model.

The updated frequencies are reported in the last row of *Tab. 1*. While the remaining errors are still present, they are considered acceptable. These residual discrepancies are likely due to the fact that the model was optimized by adjusting only a single factor. Consequently, a homogeneous reduction of stiffnesses is not sufficient to precisely match all experimental data. Other factors, such as local variations in material properties, imperfections in block connections, or modeling assumptions, may contribute to these remaining differences.

The updated model has been employed to compute the mode shapes of the Colosseum, which are depicted in *Fig. 10* alongside the experimentally determined modes. Since the experimental modes are only available at instrumented points, the comparison is somewhat limited, yet still insightful. Overall, the comparison is quite satisfactory. The deformed shapes are qualitatively similar, with both numerical and experimental results showing that the external wall experiences the largest displacement, particularly at the free ends.

The identification of the first and second modes is quite accurate. These modes are primarily bending modes localized at Stern's and Valadier's buttresses, which emerge as the most vulnerable parts of the Colosseum, because they are the terminal portions of the interrupted external wall and are not retained by radial walls.

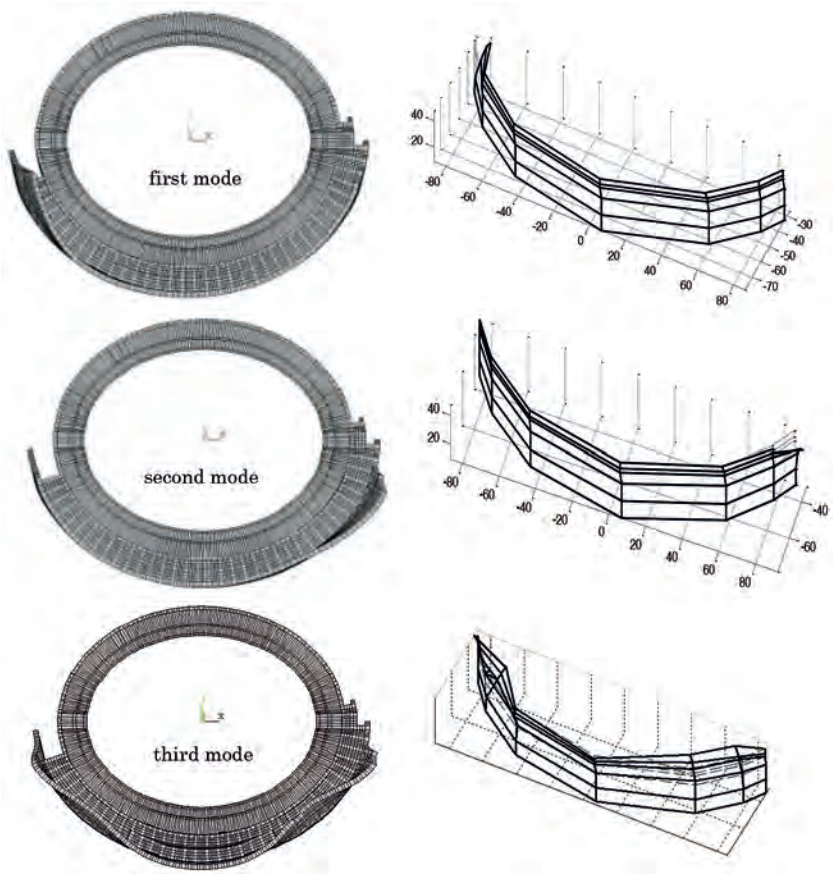

Fig. 10. Numerical modes (left column) and experimental modes of the instrumented zone (right column).

As the mode order increases, the external wall motion becomes more global, involving bending deformations in the horizontal planes. This behavior is particularly noticeable in the top displacements of the wall, indicating a broader participation of the structure in the oscillations. The comparison of the third mode between experimental and numerical results is again satisfactory, showing good agreement in the deformed shapes (*Fig. 10*). However, the fourth experimental mode does not correspond to any of the numerical modes, suggesting a discrepancy that may be attributed to localized structural complexities or model limitations. On the other hand, there is a notable similarity between the fifth and sixth experimental modes and the fourth and fifth analytical modes, indicating that the model is generally accurate but may require further refinement.

Despite the slight observed discrepancies, the updated finite element model proves sufficiently accurate for analyzing the dynamic behavior of the Colosseum and monitoring its structural health. By regularly evaluating the modal quantities and identifying corresponding models over time, variations in mechanical properties can be detected, which may indicate the onset of structural damage.

Recent vibration data collected using accelerometers installed for the construction of the Metro C Line has provided an opportunity to re-identify the monument modal quantities. These new measurements, obtained in 2022, were compared with those from the 2006 experimental campaign, and the results are practically identical. Specifically, the frequencies identified in both experimental campaigns are nearly the same, as shown in *Tab. 2*. This consistency in modal quantities over a 16-year period indicates that no significant changes in the structural mechanical properties have occurred, confirming that the Colosseum has experienced no appreciable damage between 2006 and 2022.

Tab. 2. Frequencies of first modes (Hz) obtained from records of 2006 and 2022 experimental campaign.

Mode	**1**	**2**	**3**	**4**	**5**	**6**
Frequency [10}	1.03	1.30	1.49	1.60	1.66	1.75
Frequency (this study)	1.04	1.28	1.47	1.62	1.71	-

It is important to note that the experimental investigation has revealed that the structure is much more flexible than anticipated, with the overall elastic modulus significantly lower than that of the specific material. As a result, the period of the monument is longer than expected, which provides a significant advantage in terms of its seismic vulnerability, as will be discussed later.

4.1 Traffic vibration

The accelerometric monitoring system, installed in 2014 and still operational, was initially introduced to monitor vibration levels during the construction of the underground line. This system also allows for the analysis of the measured vibration levels and peak values related to the effects of road traffic.

The velocity has been recorded at the base and the top of the structure in two directions over one month (January 2020), during a period of construction inactivity, when road traffic was the primary source of vibration. At the base, the maximum values in the radial and circumferential directions are approximately 0.15 mm/s. The effective values, which are a synthetic measure of signal intensity, are much lower, at 0.04 mm/s and 0.02 mm/s, respectively.

The dynamic response of the structure amplifies these values at the top, where the maximum velocities reach approximately 0.5 mm/s, with effective values of 0.15 mm/s and 0.08 mm/s in the radial and circumferential directions, respectively. The vibration levels are very similar to those measured during the 2006 campaign and, more importantly, they do not exceed the limit values set by international codes [12], ensuring that there is no risk of structural degradation.

4.2 Seismic vulnerability

As is well known, Italy is a country with a significant risk of earthquakes. Throughout the centuries, numerous earthquakes have struck the region, some of which have caused severe destruction. Even Dante alludes to the phenomenon in Inferno (Canto III, Verses 130-136):

Finito questo, la buia campagna
tremò sì forte, che de lo spavento
la mente di sudor ancor mi bagna.

La terra lagrimosa diede vento,
che balenò una luce vermiglia
la qual mi vinse ciascun sentimento;
e caddi come l'uom cui sonno piglia.

Of course, the seismic risk is not uniform across the country; different areas are classified by increasing expected earthquake intensity, ranging from Zone I to Zone IV. Rome is situated in Zones II and III, which corresponds to medium-low seismic intensity. An earthquake acts as an acceleration imposed at the base of the structure, equivalent to a system of forces applied to the structure, which are proportional to the masses and base acceleration and alternate with positive and negative signs, with an average close to zero.

As is typical with dynamic forces, their effect depends not only on their intensity, as is the case with static forces, but more importantly on the similarity between the dynamic characteristics of the excitation and those of the structure. This relates to the phenomenon of resonance, where a small force can produce a large response when the excitation frequencies are close to the natural frequencies of the structure. Additionally, the impact of an earthquake depends on how closely the force distribution matches the shape of the structural modes. Since the seismic forces at every moment are directed in the same direction, their distribution is most effective in the shape of the lower modes, which exhibit displacements in the same direction. For example, the first mode of the structure shown in *Fig. 6* plays the most significant role in the response, while the contributions of higher modes progressively diminish - this is a general observation in structural dynamics.

Earthquakes characterized by rapid temporal variations and high frequencies (or short periods) are not dangerous for highly flexible structures, which oscillate at low frequencies, like the first two shown in *Fig. 11*. However, such earthquakes pose a significant danger to rigid structures, like the last two shown in the figure. Unfortunately, the Colosseum falls into the latter category of rigid structures, and its seismic risk requires further investigation.

Fig. 11. Examples of flexible structures (left) and rigid structures (right).

Let us consider two simple frame structures, shown in *Fig. 12a*, with different dynamic characteristics: Frame A is rigid, with a short period, as evidenced by the fast time history of its free oscillations, shown in the zoomed section. Frame B, on the other hand, is flexible, with a long period, and its free oscillations are much slower, as clearly depicted in the related zoom. When the same seismic accelerogram (*Fig. 12b*), representing fast shaking, is applied to their bases, the effects on the two structures are quite different, as shown in *Fig. 12c*. The response of Frame A is much larger than that of Frame B because its dynamic characteristic - a short natural period - aligns with the fast components of the accelerogram. In contrast, Frame B, with a longer natural period, is less affected by these components. As a result, the maximum response displacement of Frame A is more than twice that of Frame B.

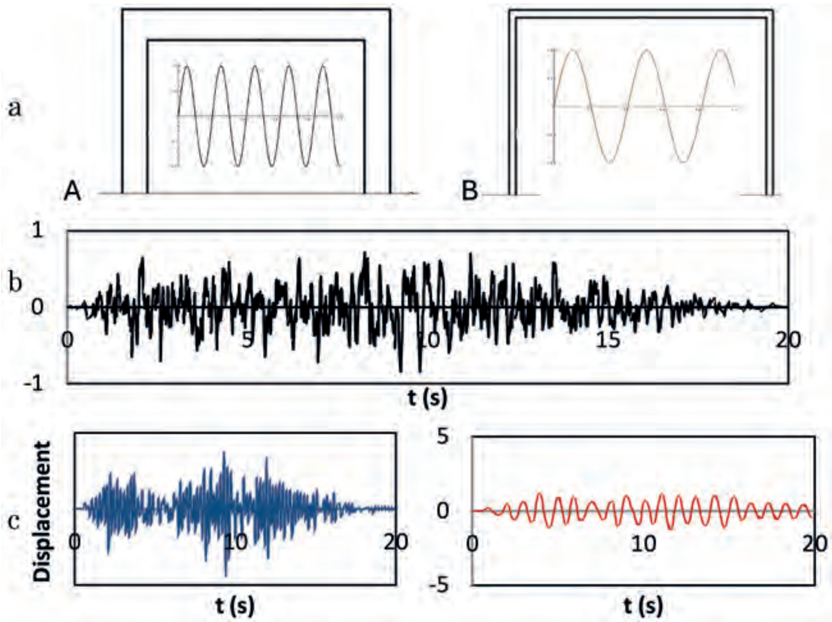

Fig. 12. (a) Earthquake accelerogram; (b) rigid and flexible frames; (c) time-history of the response of frame A and frame B to the earthquake.

Under the assumption of a linear behaviour, for a given family of earthquakes expected in a seismic zone, it is possible to calculate the maximum effect, such as the maximum acceleration, experienced by a structure based on its natural period. The result is shown in *Fig. 13* as the earthquake response spectrum in terms of acceleration. It is clear that the effect is greatest in the region of short periods, where rigid structures, like most masonry buildings, are located. The effect is moderate in the intermediate range, where framed structures with 6 to 12 floors are found, and it is minimal in the high-period zone, where flexible, slender structures are located.

It is worth noting that the first period of the Colosseum, equal to 0.43 s, determined based on literature values of material properties, falls within the range where earthquake effects are significant. However, the actual period determined from experimental results is higher, 0.97 s, placing it in a range where earthquakes are less hazardous, an encouraging outcome of this research (*Fig. 13*).

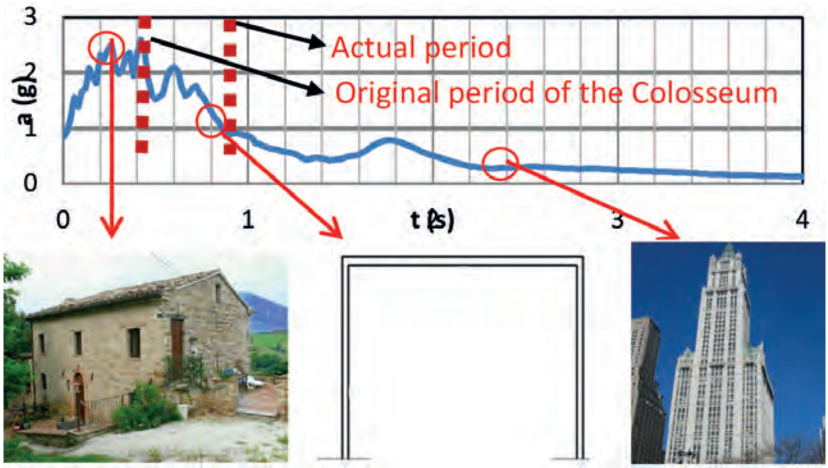

Fig. 13. (a) Earthquake Response Spectrum; (b) different effect for three samples of structure categories: rigid, frame, slender structures.

The accelerometric system installed on the monument recorded oscillations caused by recent earthquakes in Central Italy. Although the earthquake epicenters were nearly 100 kilometers from the Colosseum, resulting in mild shaking, analyzing these measurements provides a unique opportunity to understand the structure behaviour and test our predictive models [13,14]. Unfortunately, the current setup lacks instruments on the two external buttresses, which are likely the most critical parts of the structure, limiting the information we can obtain about them.

Referring to the earthquake event of October 30, 2016, which had a magnitude of 6.5, the response spectrum of the various components, shown in *Fig. 14*, indicates that the earthquake was particularly strong in the period range of 0.4 to 0.8 seconds. The periods of the first six modes of the monument are also plotted in the figure. It is a favorable outcome that the first two periods are largely outside this range, as previously noted, while the remaining four modes fall within it and are potentially affected by the seismic excitation. However, as explained earlier, higher modes (see *Fig. 10*) are less excited by uniformly distributed forces, as the seismic loads at each time instant, and therefore have a negligible impact on the overall response.

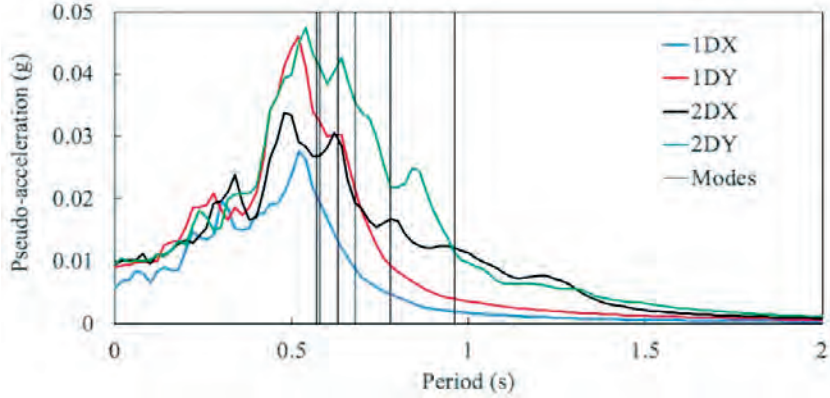

Fig. 14. Response spectra of the seismic event of October 30, 2016.

Based on the instrument setup shown in *Fig. 9*, the deformed shapes with maximum oscillation amplitude in the monument monitored portion were obtained from measurements during the seismic event. The deformations in the circumferential and radial directions are represented in *Fig. 15*. It is clear that the most affected area is the third level, which largely lacks the support from the lower radial structures in restraining the free flexion, particularly in the radial direction.

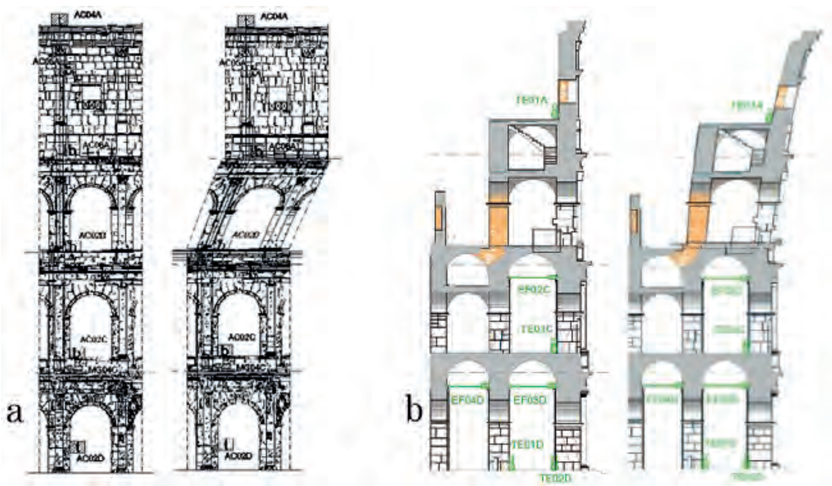

Fig. 15. Drift maximum amplitude in circumferential (a) and radial directions (b) recorded during the seismic event of October 30, 2016.

The results obtained from the recorded accelerations induced by an actual earthquake offer invaluable insights into the monument vulnerability. In the plane of the external wall (circumferential direction), a drift of 0.34 ‰ was measured between two levels, alongside a similar out-of-plane drift (radial direction), both indicating flexion in the columns of the third level in both directions. These values were measured during an earthquake recorded near the base of the monument with a peak ground acceleration of approximately 0.01g. In contrast, an expected earthquake in Rome, with a return period of 975 years, is characterized by a peak ground acceleration of around 0.14g. Assuming linear structural behaviour, this would result in a drift of 4.74 ‰. However, this value may be reduced by a factor of 1.5 to 2 due to the structure nonlinear behaviour, leading to an expected drift of about 3 ‰, which can be considered within acceptable limits.

Nonetheless, this preliminary evaluation of the seismic risk of the Colosseum necessitates a more sophisticated approach. This approach involves selecting acceptable protection levels and analyzing response data obtained from a reliable validated model for different earthquakes. Beyond assessing the overall behaviour, it is crucial to investigate local mechanisms, particularly at the top of the outer wall, where significant accelerations may occur.

The data collected from recording the monument response to future strong motions will be invaluable for understanding its dynamic behaviour and refining the predictive model to better assess the monument vulnerability. In this context, the current sensor setup should be expanded to cover critical areas that lack instrumentations, as identified by the numerical model, prioritizing locations with maximum displacement in the modal shapes.

5. Conclusions

The Colosseum was erected nearly two thousand years ago. The magnificent elliptical structure, originally designed, was highly resistant against both vertical and horizontal forces. However, foundation settlements, along with several earthquakes, have led to the collapse of significant portions of the external wall on the south side, making the structure more vulnerable, particularly to horizontal actions. Today, with modern knowledge and expertise, it is both possible and essential to preserve the structure in its current state and prevent any further collapses.

A detailed description of the mechanical characteristics of the structure can be provided and accurate models capable of simulating its dynamic behaviour have been developed. The ability to easily determine the structure dynamic properties - such as natural frequencies and modes - based on measured responses to ambient excitations, like traffic, wind, and micro-earthquakes, is crucial for implementing an effective dynamic monitoring system. In this approach, any changes in the dynamic properties over time are observed, as they can indicate potential damage. Moreover, if the experimental data is accurate and the model is well-calibrated, the solution of an inverse problem can yield reliable insights into location and severity of damage.

As an example, a comparison of the dynamic properties obtained from two experimental campaigns conducted in 2006 and 2020 showed no significant variations in the structural characteristics, indicating that no appreciable damage occurred in the structure during this time interval.

The study concludes by discussing the effects of road traffic and potential earthquakes, both based on experimental recordings. The velocity values measured at the base and top of the monument due to traffic, across various investigations, remain within the limits set by international codes. The analysis confirms that traffic-induced vibrations are not of sufficient amplitude to cause structural damage. However, this phenomenon must be monitored over a long period to ensure ongoing safety.

Regarding seismic vulnerability, the measured response of the structure to a magnitude 6.5 earthquake during the 2016 seismic sequence in Central Italy has provided preliminary insights into the potential effects of an earthquake of the expected intensity in Rome. The initial conclusions are cautiously optimistic, but an accurate assessment of the monument seismic risk requires a reliable model and additional experimental data. The information gained from recording the monument response to future strong motions is important for enhancing our understanding of its dynamic behaviour and updating the predictive model used to assess its vulnerability more accurately. To achieve this, the current sensor setup should be enlarged to cover some critical areas, identified by numerical investigations, which presently lack sensor coverage.

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